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and

$$\angle\, CBD = \tfrac{1}{2}(B - C).$$

2. Draw AP, the perpendicular bisector of BD, and produce it to meet BC at Q; then

$$AP = BP \cdot \tan \tfrac{1}{2}(B + C);$$

and

$$PQ = BP \cdot \tan \tfrac{1}{2}(B - C).$$

3. Draw PR parallel to BC, bisecting CD at R; then

$$AR = \tfrac{1}{2}(b + c);$$

and

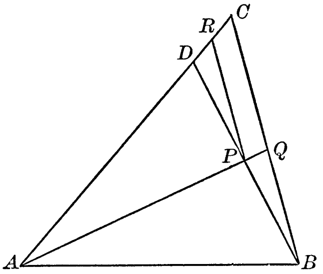
$$RC = \tfrac{1}{2}(b - c).$$

4. But

$$\frac{AR}{RC} = \frac{AP}{PQ};$$

therefore

$$\frac{b + c}{b - c} = \frac{\tan \tfrac{1}{2}(B + C)}{\tan \tfrac{1}{2}(B - C)}.$$



QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

DISCUSSIONS.

Professor J. E. Trevor gave an example in the December number of the MONTHLY of an instance in which a theorem of analysis is of importance in thermodynamics. Another case of this sort is contributed by him as the first discussion in the present number. Corresponding to a change of independent variable, the form of a function is altered; it is readily shown that the rate of change can be represented as an ordinary derivative of another function closely related to the first. The notion is of use in connection with the heats of dilution of a solution. Instances in which mathematical conceptions arise in connection with applied science are always of value; even those mathematicians whose personal interests are principally in so-called "abstract" fields cannot afford to neglect the phases of mathematics sometimes termed "practical."

In the study of analytic geometry beyond the more elementary portions determinants play an extremely important part; a glance at any treatise on projective or metric analytic geometry, on differential geometry, or on non-Euclidean geometry analytically treated, will reveal page after page filled with resultants, discriminants, Hessians, linear dependence, and other ideas based on determinants. In the elementary parts of the subject, however, determinants play a negligible rôle. In most texts in common use, the determinantal form for the area of a triangle is a solitary instance. Professor Foraker in the second discussion below gives an introduction to the use of determinants in the elementary field. Most of his work consists of the statement of formulae; one theorem is proved, relating to the collinearity of the circumcenter, centroid, and orthocenter of a triangle. It is scarcely to be expected that the material would be suitable for ordinary class use, but it will be of interest to a few students in any class.

The third discussion is a plea for the more general use of the history of mathematics in connection with secondary and collegiate instruction. Attracted by an announcement of the plans of the National Committee on Mathematical Requirements, Mr. Laurin Zilliacus of the Bedales School in Petersfield, England, has written a letter on the subject to Professor H. W. Tyler, a member of the committee. With the permission of the writer, the recipient, and Professor J. W. Young, chairman of the committee, we reproduce nearly the entire letter, retaining the personal form in which it is written. The fact that Mr. Zilliacus has actual experience of the growth of interest and enthusiasm in a class through the use of historical material is of more value than any amount of mere theorizing on the question.

Another article basing pedagogic recommendations on the result of personal experience is found in the last discussion, by Professor Ettlinger, on the use of graphical methods in trigonometry. It is nearly certain that all teachers make some use, in their teaching, of the ideas suggested by Professor Ettlinger; not all, however, have carried these ideas so far, or treated them with such emphasis. One incidental point in the paper moves the editor to a comment which he has long hoped to see made. It is pointed out that a correctly drawn figure is of great aid in attacking a problem. This is undeniably true in trigonometry, and especially in elementary geometry. Is it not also true, on the other hand, that sometimes a figure drawn with carefully planned *inaccuracy* is of extreme importance and inspiration?

I. HEATS OF DILUTION.

By J. E. TREVOR, Cornell University.

The "heats of dilution" of a solution are mathematical curiosities, in that these quantities are defined with reference to changes of the form of a function. When $\psi(x)$ is a given function, and δx is an arbitrary positive increment of the independent variable x, let it be supposed that a certain operation changes the value of a quantity φ from the value

$$\varphi_1 = \psi(x) + a \cdot \delta x$$

to the value

$$\varphi_2 = \psi(x + \delta x),$$

where a is a constant. The change of value $\varphi_2 - \varphi_1$ is due to the change of